Ingegneria delle Telecomunicazioni

Satellite Communications

19. Cold and Warm – Ranging Code Acquisition and Tracking

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TOA Measurement: Basics of Estimation Theory

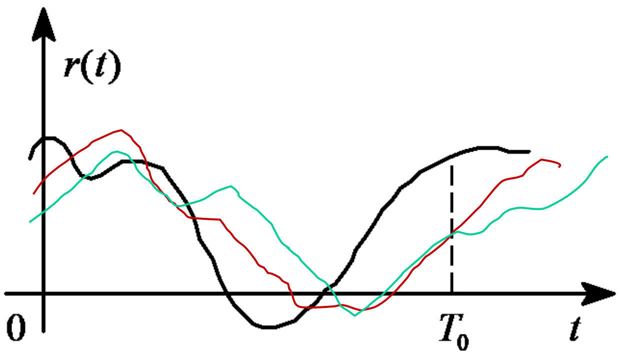
Available observation: the received waveform $r(t;\tau), \quad 0 \le t < T_0$

***Estimate** of the parameter :* $\hat{\tau} = F\left[r(t;\tau)\right]$

*Estimator **bias*** $\beta_{\hat{\tau}} = E\left\{\hat{\tau}\right\} - \tau$

*Estimator **Variance*** $\sigma_{\hat{\tau}}^2 = E\left\{\left(\hat{\tau} - E\left\{\hat{\tau}\right\}\right)^2\right\}$

*Estimator **MSE*** $MSE(\hat{\tau}) = E\left\{\left(\hat{\tau} - \tau\right)^2\right\} = \sigma_{\hat{\tau}}^2 + \beta_{\hat{\tau}}^2$



Because of noise, the estimate is a random variable; its accuracy (i.e., its MSE), depends on the amount of noise or on the SNR !

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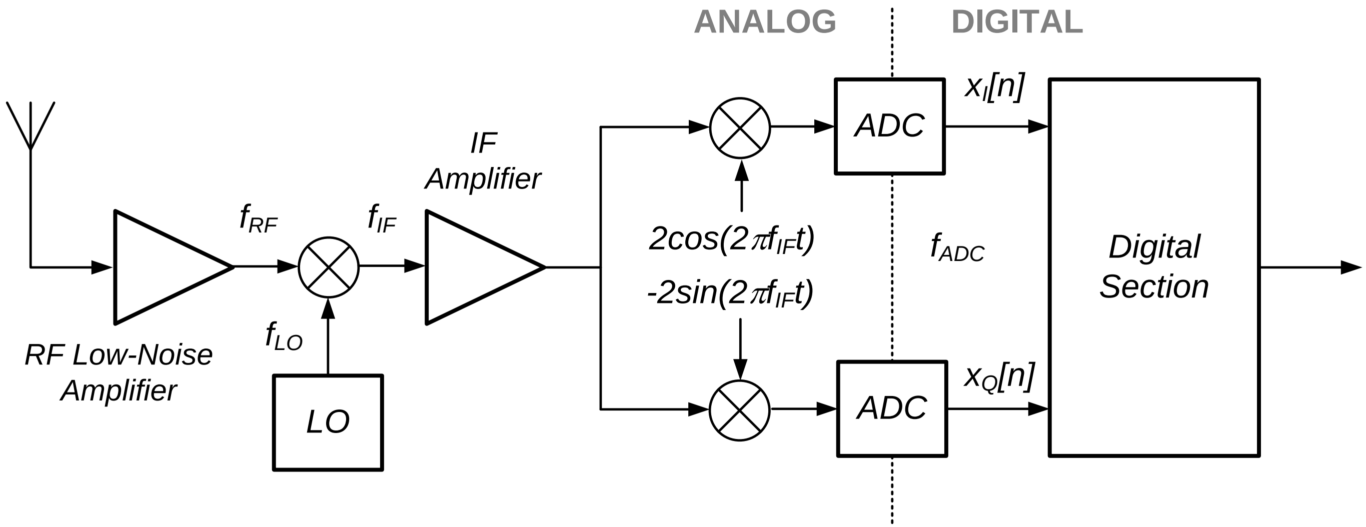
Receiving the GNSS Signal…

- **Standard I/Q Radio receiver with Digital Signal Processing**



Basic Modeling of the I/Q received signal

- N_{sat} : number of satellites in visibility *(elevation larger than ≈10 degrees)*
- i : satellite identifier
- C_i : received signal power
- τ_i : time-of-flight in the user time scale (group delay)
- Δf_i : carrier Doppler shift (plus local oscillator frequency offset)
- θ_i : satellite carrier phase shift

$$r(t) = \sum_{i=1}^{N_{sat}} \sqrt{2C_i}\, s_i(t-\tau_i) \exp\left[j\left(2\pi\Delta f_i t + \theta_i\right)\right] + w(t)$$

Let us super-simplify and consider just the delay from 1 satellite as the only parameter

The (Modified) Cramér-Rao Bound

*The (**Modified) Cramér-Rao Bound** gives the best accuracy that can be attained by **any** unbiased estimator*

$$r(t) = s(t;\tau,\mathbf{c}) + w(t)$$

desired parameter (τ)

Vector of "side" parameter(s): chips of the ranging code considered UNKNOWN and random ($\mathbf{c}$)

$$E\left\{\left|\hat{\tau}-\tau\right|^2\right\} \geq \frac{N_0}{E_{\mathbf{c}}\left\{\int_0^{T_0}\left|\frac{\partial s(t;\tau,\mathbf{c})}{\partial \tau}\right|^2 dt\right\}}$$

The MCRB for pseudorange accuracy 1/2

$$\sigma_\tau[\mathrm{m}] \geq cT_c \times \sqrt{\frac{B_L}{2C/N_0}} \left(\frac{1}{2\pi\beta} \right)$$

$B_L = 1/(2T_0)$ *loop bandwidth (we'll see later on what it means) [Hz]*

C/N_0 *signal-to-noise-ratio per unit bandwdith [dB · Hz]*

cT_c *equivalent chip length [m]*

$S_s(f)$ *GNSS signal psd*

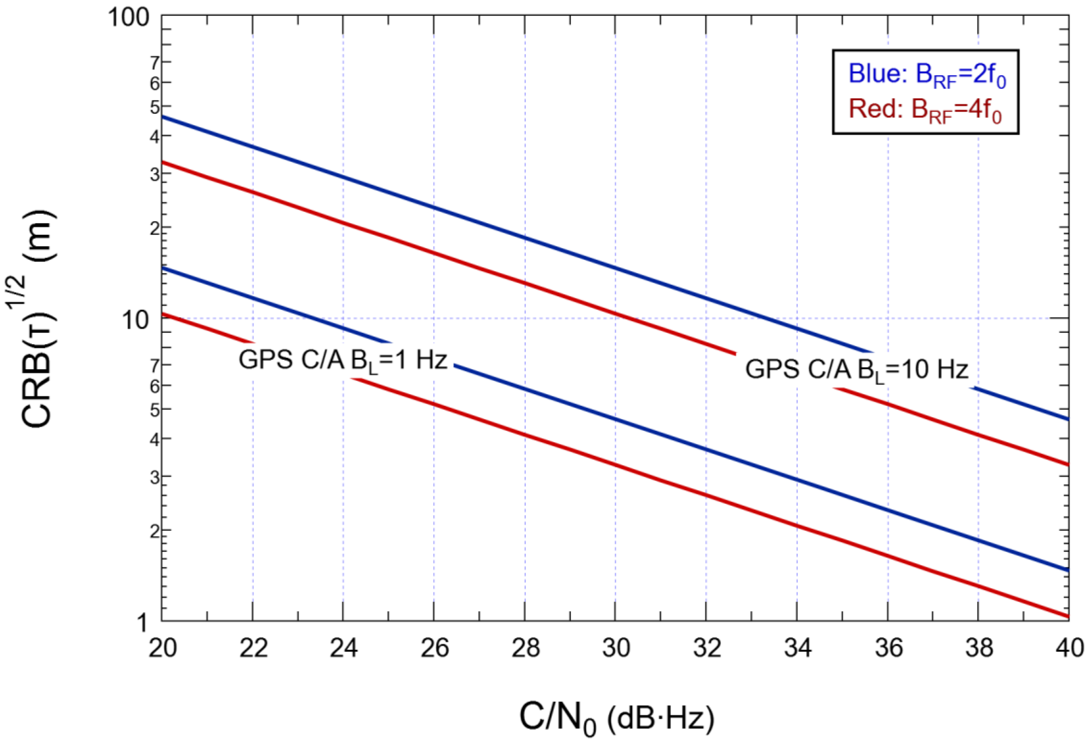
$$\beta^2 = \frac{T_c^2 \int_{-B_R F/2}^{+B_{RF}/2} f^2 S_s(f)\, df}{\int_{-B_{RF}/2}^{+B_{RF}/2} S_s(f)\, df}$$

Normalized Squared Gabor Bandwidth in the receiver (radio) bandwidth B_{RF}

The MCRB for pseudorange accuracy 2/2

$$\sigma_\tau[\mathrm{m}] \geq cT_c \times \sqrt{\frac{B_L}{2C/N_0}}\left(\frac{1}{2\pi\beta}\right)$$

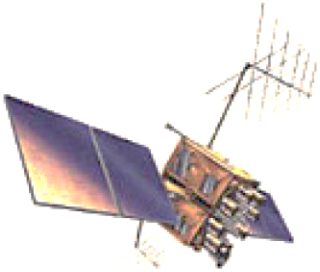
$$\beta^2 = \frac{T_c^2 \int_{-B_{RF}/2}^{+B_{RF}/2} f^2 S_s(f)\,df}{\int_{-B_{RF}/2}^{+B_{RF}/2} S_s(f)\,df}$$

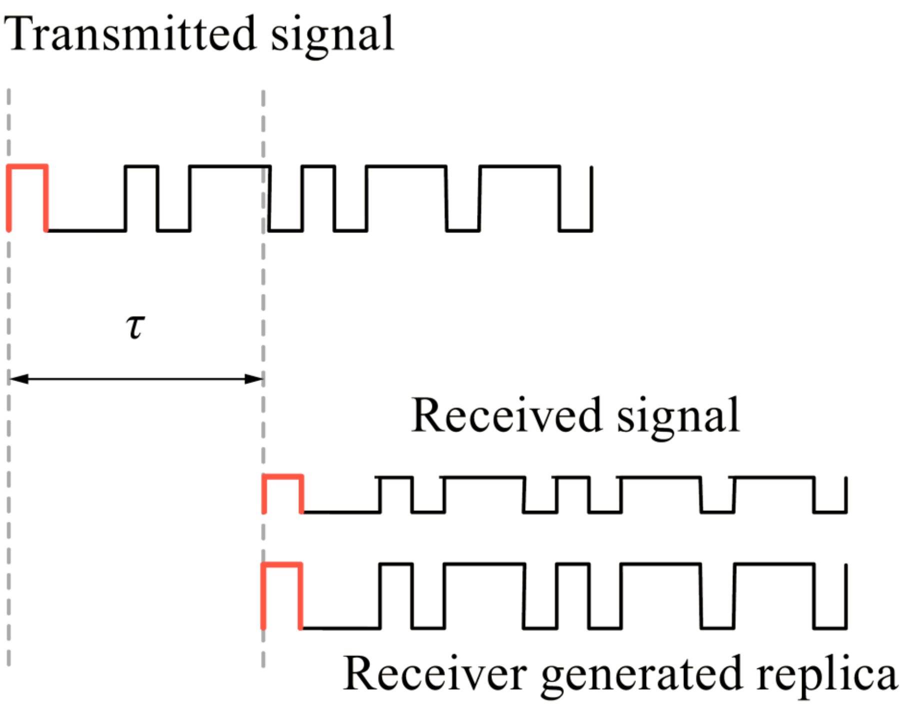


For usual values of loop bandwidth and SNR, the RMS error due to receiver noise is smaller than 1 m

How far is the Bound? How can we actually estimate the TOA?

- The main idea is simple: implement in the receiver a local ranging code generator, and try to «lock» the local code to the code into the received signal, then evaluate τ based on the local clock of the locked code generator. Is it an *optimal* criterion?



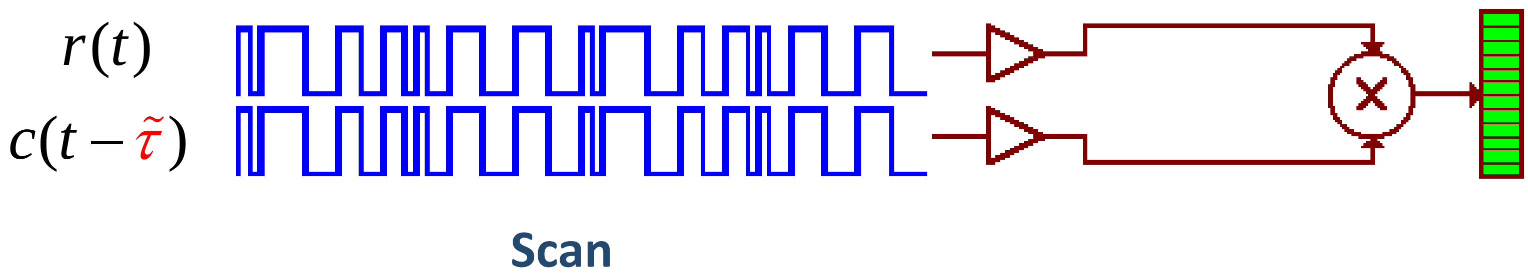


Optimum (Maximum-Likelihood) code acquisition

Simple correlation processing on the observation time T_0:

$$\hat{\tau} = \arg\max_{\tilde{\tau}} \left\{ \int_0^{T_0} r(t) c(t-\tilde{\tau}) dt \right\}$$

Scan the delay values $\tilde{\tau}$ and find that one that maximizes the cross-correlation between the received signal r and the local replica code c



The main issue in TOA estimation

- *GPS C/A code repetition period LT_c : 1 ms (1023 chips)*
- *Typ. required estimation accuracy σ_τ : 3 m, i.e., 10 ns*
- *$\sigma_\tau / LT_c = 10^{-5}$ very, very high relative accuracy – on average, it needs $10^5/2$ tries (scan steps)!*
- *The solution: Need to break-up estimation into COARSE and FINE estimation, or, ACQUISITION and TRACKING*
  - *Tracking comes after acquisition is accomplished and provides the small final required accuracy*
  - *Acquisition (aka COLD Acquisition) provides an initial coarse accuracy of $T_c/2$, requiring much less scan steps, then hands over to tracking*

Once acquisition is over…

To maximize $\int_0^{T_0} r(t)c(t-\tilde{\tau})dt$ ***we do:***

$$\frac{d}{d\tilde{\tau}}\int_0^{T_0} r(t)c(t-\tilde{\tau})dt = 0 \Rightarrow \int_0^{T_0} r(t)\frac{d}{d\tilde{\tau}}c(t-\tilde{\tau})dt = 0$$

$$\frac{d}{d\tilde{\tau}}c(t-\tilde{\tau}) \cong -\frac{c(t-\tilde{\tau}+\Delta)-c(t-\tilde{\tau}-\Delta)}{2\Delta} \Rightarrow \int_0^{T_0} r(t)\frac{c(t-\tilde{\tau}+\Delta)-c(t-\tilde{\tau}-\Delta)}{2\Delta}dt = 0$$

i.e., we have to solve

$$\int_0^{T_0} r(t)c(t-\tilde{\tau}+\Delta)dt - \int_0^{T_0} r(t)c(t-\tilde{\tau}-\Delta)dt = 0$$

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… fine tracking is started…

$$e(\tilde{\tau}) \triangleq \int_0^{T_0} r(t)c(t-\tilde{\tau}+\Delta)dt - \int_0^{T_0} r(t)c(t-\tilde{\tau}-\Delta)dt$$

Early correlation

Late correlation

Iterative Solution @ T_0 steps

$$\tilde{\tau}[n+1] = \tilde{\tau}[n] - \gamma e(\tilde{\tau}[n])$$

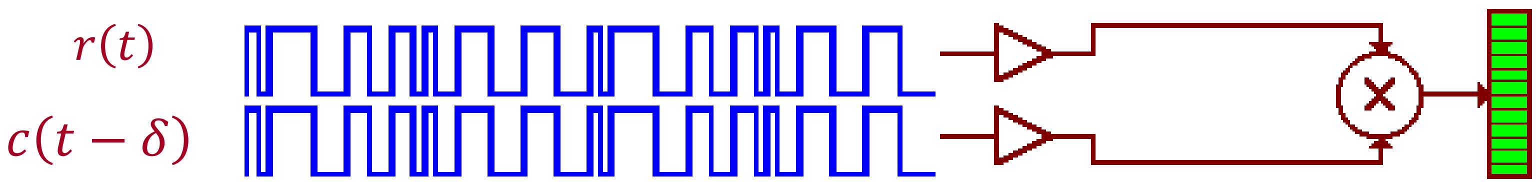
Delay-Lock Loop (DLL) (to be continued)

In reality…

Simple correlation processing:

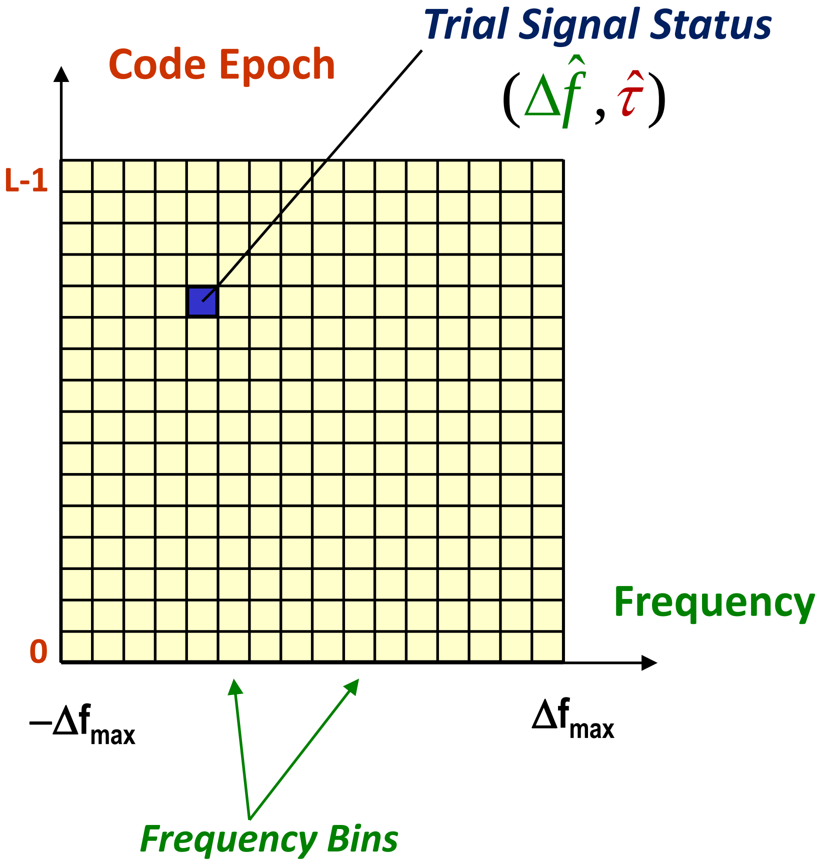
$$\hat{\tau} = \arg\max_{\tilde{\tau}} \left\{ \int_0^{T_0} \underbrace{c(t-\tau)\exp\left[j(2\pi\Delta f t + \theta)\right]}_{\text{pilot received signal}} c(t-\tilde{\tau})\,dt \right\}$$

It's not so simple because of the presence of the Doppler shift and of an arbitrary phase-shift – processing has to be I/Q and non-coherent (i.e, independent of the carrier phase)



Scan

Initial Code/Frequency Acquisition



Bidimensional Search:

- *The scan has to be carried out both on frequency offset and on code delay (epoch), and it takes time…*
- *It has to be **noncoherent** (squared modulus of cross-correlation)*
- *It can be sped up using FFTs*

$$(\Delta\hat{f}, \hat{\tau}) = \arg\max_{\Delta\tilde{f}, \tilde{\tau}} \left\{ \left| \int_0^{T_0} r(t) \exp\left[-j2\pi\Delta\tilde{f}\, t\right] c(t-\tilde{\tau})\, dt \right|^2 \right\}$$

Typical 2D Acquisition